String Theory Survey

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According to Einstein's theory, a relativistic equation has to use coordinates that have the proper Lorentz transformation properties. But then we have a problem, because a string oscillates in space and time, and as it oscillates, it sweeps out a two-dimensional surface in spacetime that we call a world sheet (compared with the world line of a particle).

In the nonrelativistic string, there is a clear difference between the space coordinate along the string, and the time coordinate. But in a relativistic string theory, we wind up having to consider the world sheet of the string as a two-dimensional spacetime of its own, where the division between space and time depends upon the observer. The classical equation can be written as

$$\frac{\partial^2 X^\mu(\sigma,\tau)}{\partial\tau^2} = c^2 \frac{\partial^2 X^\mu(\sigma,\tau)}{\partial\sigma^2}$$

where s and t are coordinates on the string world sheet representing space and time along the string, and the parameter c^2 is the ratio of the string tension to the string mass per unit length.

These equations of motion can be derived from Euler-Lagrange equations from an action based on the string world sheet

$$S = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{h} h^{nm} \partial_m X^\mu \partial_n \mu$$

The spacetime coordinates Xm of the string in this picture are also fields Xm in a two-dimension field theory defined on the surface that a string sweeps out as it travels in space. The partial derivatives are with respect to the coordinates s and t on the world sheet and hmn is the two-dimensional metric defined on the string world sheet.

The general solution to the relativistic string equations of motion looks very similar to the classical nonrelativistic case above. The transverse space coordinates can be expanded in normal modes as

$$X^i(\sigma,\tau) = x^i + \dot{x}^i\tau + i\sqrt{2\alpha'} \sum_{n=0}^{\pm\infty} \frac{1}{n} \alpha_n^i \left(\cos\frac{n\pi c\tau}{L} - i\sin\frac{n\pi c\tau}{L}\right)\cos\frac{n\pi\sigma}{L}$$

The string solution above is unlike a guitar string in that it isn't tied down at either end and so travels freely through spacetime as it oscillates. The string above is an open string, with ends that are floppy.

For a closed string, the boundary conditions are periodic, and the resulting oscillating solution looks like two open string oscillations moving in the opposite direction around the string. These two types of closed string modes are called right-movers and left-movers, and this difference will be important later in the supersymmetric heterotic string theory.

This is classical string. When we add quantum mechanics by making the string momentum and position obey quantum commutation relations, the oscillator mode coefficients have the commutation relations

$$[\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n}\eta^{\mu\nu}$$

The quantized string oscillator modes wind up giving representations of the Poincaré group, through which quantum states of mass and spin are classified in a relativistic quantum field theory.

So this is where the elementary particle arise in string theory. Particles in a string theory are like the harmonic notes played on a string with a fixed tension

$$T_{\text{string}} = \frac{1}{2\pi\alpha'}$$

The parameter α' is called the string parameter and the square root of this number represents the approximate distance scale at which string effects should become observable.

In the generic quantum string theory, there are quantum states with negative norm, also known as ghosts. This happens because of the minus sign in the spacetime metric, which implies that

$$\left[\alpha_m^0, \alpha_n^0\right] = -m\delta_{m+n}$$

So there ends up being extra unphysical states in the string spectrum. In 26 spacetime dimensions, these extra unphysical states wind up disappearing from the spectrum. Therefore. bosonic string quantum mechanics is only consistent if the dimension of spacetime is 26.

By looking at the quantum mechanics of the relativistic string normal modes, one can deduce that the quantum modes of the string look just like the particles we see in spacetime, with mass that depends on the spin according to the formula

$$J = \alpha' M_j^2$$

Remember that boundary conditions are important for string behavior. Strings can be open, with ends that travel at the speed of light, or closed, with their ends joined in a ring.

One of the particle states of a closed string has zero mass and two units of spin, the same mass and spin as a graviton, the particle that is supposed to be the carrier of the gravitational force.