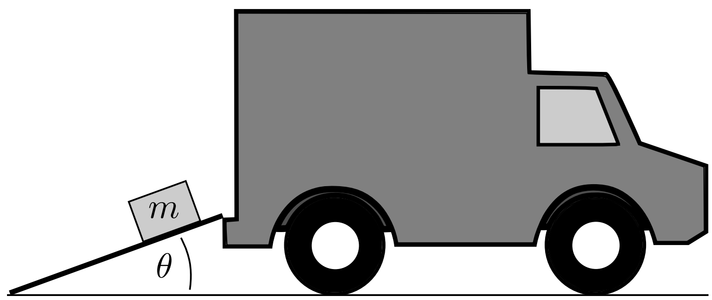
Classical Mechanics Problem - Non-conservative forces - Friction of box on an incline

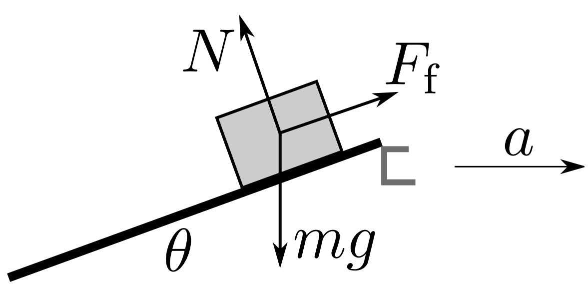
March 19, 2012

A box of mass m rests on a delivery truck ramp that is slanted at an angle θ from the ground.

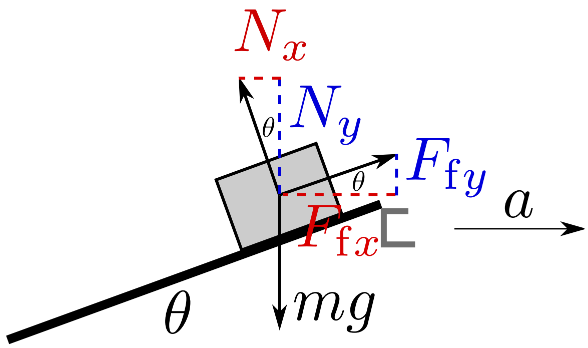


The coefficient of static friction between the box and the ramp is μ_s. The ramp is hooked to the truck and slides across the ground as the driver takes off. Find the acceleration a at which the driver must not exceed for him to not lose the box. You must find it symbollically, in the most compact form you can and argue why this form is more useful than others.

Solution: The only forces acting on the box are the normal force N, the friction force F, and the force of gravity mg. We are considering the whole system accelerating at a, so the force ma is the sum of the forces in the horizontal plane. Therefore we draw the acceleration vector outside of the box's free body diagram.



We project the forces acting on the box onto a coordinate system that simplifies the dynamics.



Now sum the forces to get the necessary information for isolating a.

$$\sum F_x = F_{\mathrm{f}x} - N_x = ma$$

$$\sum F_y = F_{\mathrm{f}y} + N_y - mg = 0$$

We may enhance the detail of the solution by projecting involves trigonometry of θ.

$$F_\mathrm{f}\cos\theta - N\sin\theta = ma$$

$$F_\mathrm{f}\sin\theta + N\cos\theta = mg$$

Also, we note that $F_\mathrm{f} = \mu_\mathrm{s} N$.

$$\mu_\mathrm{s} N\cos\theta - N\sin\theta = ma$$

$$\mu_\mathrm{s} N\sin\theta + N\cos\theta = mg$$

Subtract or add the equations

$$\begin{array}{rl} & \mu_\mathrm{s} N\cos\theta - N\sin\theta = ma \\ \pm & (\mu_\mathrm{s} N\sin\theta + N\cos\theta = mg) \\ \hline \end{array}$$

$$\mu_\mathrm{s} N\cos\theta - N\sin\theta \pm (\mu_\mathrm{s} N\sin\theta + N\cos\theta) = ma \pm mg$$

$$\mu_\mathrm{s} N\cos\theta - N\sin\theta \pm (\mu_\mathrm{s} N\sin\theta + N\cos\theta) \mp mg = ma$$

$$a = \frac{1}{m}\left(\mu_\mathrm{s} N\cos\theta - N\sin\theta \pm \mu_\mathrm{s} N\sin\theta \pm N\cos\theta \mp mg\right)$$

$$a = \frac{N}{m}\left[\mu_\mathrm{s}\left(\cos\theta \pm \sin\theta\right) \pm \cos\theta - \sin\theta \mp \frac{mg}{N}\right]$$

This is the most compact I could get because I got the coefficient of friction into one term and with the normal force factored out the terms simplify.