Quantum mechanics - Free particle

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An free electron behaves like complex plane wave, with the wavelength associated with its momentum. Show that a plane wave satisfies Schrodinger's time-independent equation with no potential.

Solution:

The general solution to a wave equation may be written using separation of variables

$$\Psi\left(x,t\right)=\psi\left(x\right)\phi\left(t\right)$$

Apply this to Schrodinger's time-dependent equation

$$i\hbar\psi\left(x\right)\frac{\partial}{\partial t}\phi\left(t\right)=\frac{1}{2m}\phi\left(t\right)\left(-i\hbar\frac{\partial}{\partial x}\right)^{2}\psi\left(x\right)$$

$$i\hbar\psi\left(x\right)\frac{\partial}{\partial t}\phi\left(t\right)=-\frac{\hbar^{2}}{2m}\phi\left(t\right)\frac{\partial^{2}}{\partial x^{2}}\psi\left(x\right)$$

$$i\hbar\frac{1}{\phi\left(t\right)}\frac{\partial}{\partial t}\phi\left(t\right)=-\frac{\hbar^{2}}{2m}\frac{1}{\psi\left(x\right)}\frac{\partial^{2}}{\partial x^{2}}\psi\left(x\right)=E$$

$$-\frac{\hbar^{2}}{2m}\frac{\partial^{2}}{\partial x^{2}}\psi\left(x\right)=\hat{H}\psi\left(x\right)=\psi\left(x\right)E$$

where $-\frac{\hbar^{2}}{2m}\frac{\partial^{2}}{\partial x^{2}}$ is the Hamiltonian for a plane wave.