Statistical mechanics - distributions - mean, variance, and standard distribution

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Given a distribution in v of

$$D = Av^2 e^{-Bv^2},$$

find the first, second moment, variance, and standard deviation in the interval 0 to ∞(A and B are real and positive)?

Solution:

This first moment is

$$\langle v \rangle = \frac{A\int_0^\infty v \cdot v^2 e^{-Bv^2} dv}{A\int_0^\infty v^2 e^{-Bv^2} dv} = \frac{2}{\sqrt{B\pi}}$$

The second moment is

$$\left\langle v^2 \right\rangle = \frac{A\int_0^\infty v^2 \cdot v^2 e^{-Bv^2} dv}{A\int_0^\infty v^2 e^{-Bv^2} dv} = \frac{3}{2B}$$

The variance is

$$\sigma^2 = \left\langle v^2 \right\rangle - \langle v \rangle^2 = \frac{1}{B}\left(\frac{3}{2} - \frac{4}{\pi}\right)$$

The standard deviation is

$$\sigma = \sqrt{\frac{1}{B}\left(\frac{3}{2} - \frac{4}{\pi}\right)}$$