Thermodynamics - Calorimetry - Hot metal in ice chest

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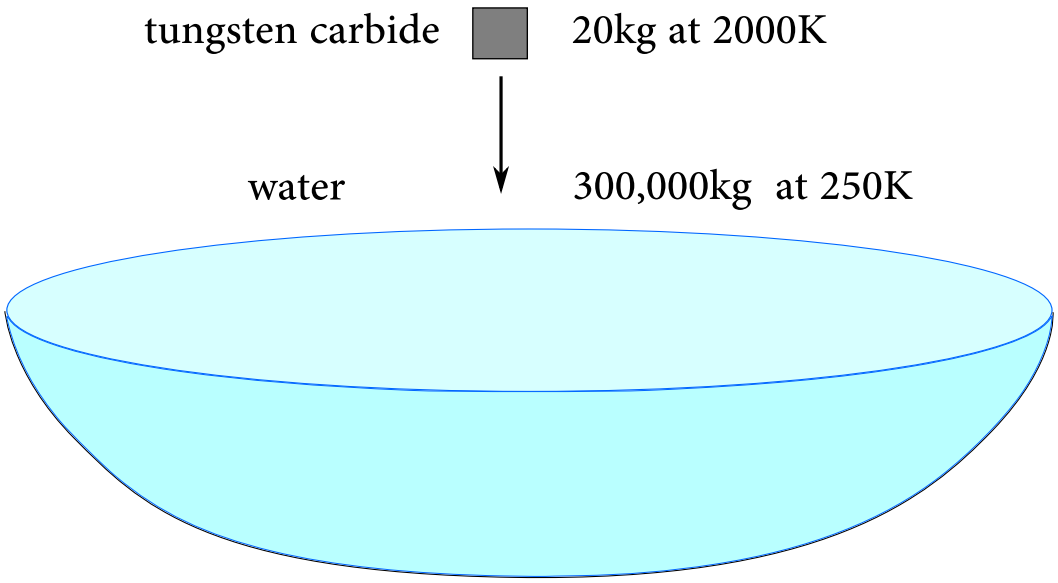
Suppose you drop a hot piece of tungsten carbide(20 kg at 2000K) into a huge lake of solid ice(300,000kg of water at 250K). What would be the final temperature of the system after a long time?



Solution:
The standard calorimetry approach begins with

$$\sum Q_i = 0$$

$$m_{\mathrm{tc}} c_{\mathrm{tc}} \triangle T - m_{\mathrm{w}} L_{\mathrm{w}} + m_{\mathrm{w}} c_{\mathrm{w}} \triangle T = 0$$

$$m_{\mathrm{tc}} c_{\mathrm{tc}} \left(T_f - T_{i\mathrm{tc}}\right) - m_{\mathrm{w}} L_{\mathrm{w}} + m_{\mathrm{w}} c_{\mathrm{w}} \left(T_f - T_{i\mathrm{w}}\right) = 0$$

$$m_{\mathrm{tc}} c_{\mathrm{tc}} \left(T_f - T_{i\mathrm{tc}}\right) - m_{\mathrm{w}} L_{\mathrm{w}} + m_{\mathrm{w}} c_{\mathrm{w}} \left(T_f - T_{i\mathrm{w}}\right) = 0$$

$$m_{\mathrm{tc}} c_{\mathrm{tc}} T_f + m_{\mathrm{w}} c_{\mathrm{w}} T_f = m_{\mathrm{tc}} c_{\mathrm{tc}} T_{i\mathrm{tc}} + m_{\mathrm{w}} c_{\mathrm{w}} T_{i\mathrm{w}} + m_{\mathrm{w}} L_{\mathrm{w}}$$

$$T_f = \frac{m_{\mathrm{tc}} c_{\mathrm{tc}} T_{i\mathrm{tc}} + m_{\mathrm{w}} c_{\mathrm{w}} T_{i\mathrm{w}} + m_{\mathrm{w}} L_{\mathrm{w}}}{m_{\mathrm{tc}} c_{\mathrm{tc}} + m_{\mathrm{w}} c_{\mathrm{w}}}$$

Get constants from trusted source. At 2000K tungsten carbide has a specific heat of about 300 $\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$. Latent heat of water fusion 334 $\frac{\mathrm{kJ}}{\mathrm{kg}}$. At 273K water has a specific heat of about 4186 $\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$.

$$T_f = \frac{(20\mathrm{kg})\left(300\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}\right)(2000\mathrm{K}) + (300,000\mathrm{kg})\left(4186\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}\right)(250\mathrm{K}) + (300,000\mathrm{kg})\left(334,000\frac{\mathrm{J}}{\mathrm{kg}}\right)}{(20\mathrm{kg})\left(300\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}\right) + (300,000\mathrm{kg})\left(4186\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}\right)}$$

$$T_f = 329\mathrm{K}$$