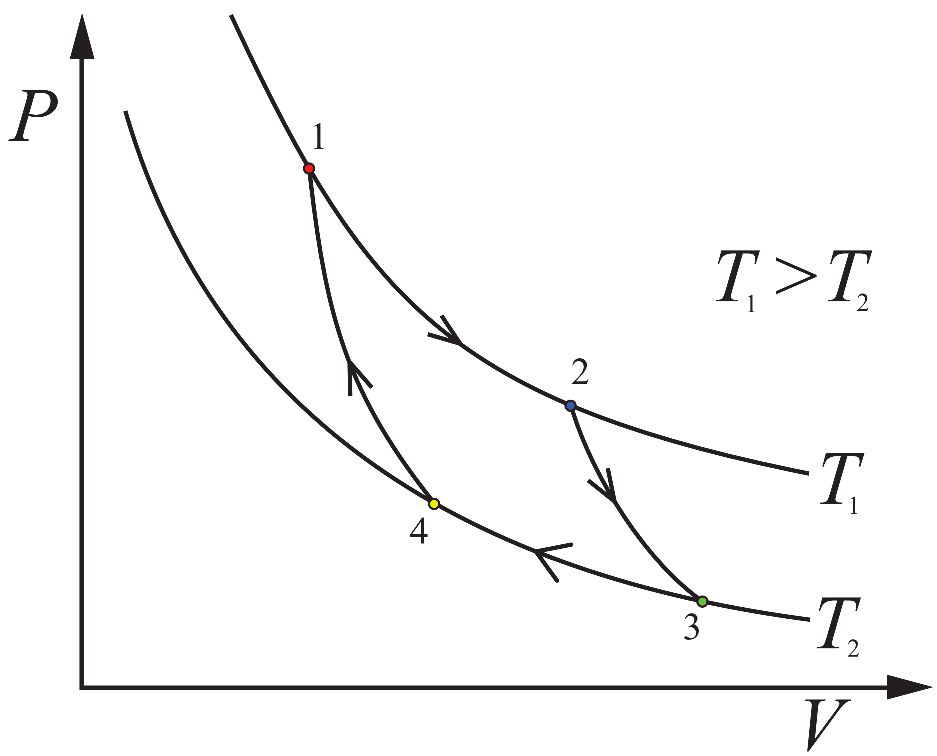
Thermodynamics - Work - The limits of a Carnot cycle

March 28, 2012

A Carnot cycle is an idealized heat engine that involves two isothermic and two adiabatic processes.



Find and draw a process with equal work output but composed of isochoric and isobaric processes where the low pressure is 1 Pa and the high is 10 Pa. Use one mole of an ideal monotomic gas as the model. Estimate the efficiency loss taking the above carnot cycle to have the values:

Cycle point	Pressures	Volumes	Temperatures
1	$P_1 = 3\text{Pa}$	$V_1 = 1\text{m}^3$	$T_1 = 7\text{K}$
2	$P_2 = 1.5\text{Pa}$	$V_2 = 2\text{m}^3$	$T_1 = 7\text{K}$
3	$P_3 = .5\text{Pa}$	$V_3 = 2.5\text{m}^3$	$T_2 = 2\text{K}$
4	$P_4 = 1\text{Pa}$	$V_4 = 1.5\text{m}^3$	$T_2 = 2\text{K}$

Solution:

In order for us to match the work output of the carnot from above we need to calculate its work output. The ideal gas law tells us that if the temperature does not change then we may express work done as

$$W_{12} = \int pdV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln\left(\frac{V_2}{V_1}\right)$$

So for the first isotherm we have

$$(1\text{mol})\left(8.314\frac{\text{J}}{\text{kgmol}}\right)(7\text{K})\ln\left(\frac{2\text{m}^3}{1\text{m}^3}\right) = 40.3398\text{J}$$

The second isotherm we get

$$W_{34} = \int_{V_3}^{V_4} \frac{nRT}{V} dV = nRT \ln\left(\frac{V_4}{V_3}\right)$$

$$(1\text{mol})\left(8.314\frac{\text{J}}{\text{kgmol}}\right)(2\text{K})\ln\left(\frac{2.5\text{m}^3}{1.5\text{m}^3}\right) = -8.49401\text{J}$$

The work for the first adiabat $4 \to 1$ is:

$$W_{\text{a1}} = \frac{1}{\gamma - 1}(P_4V_4 - P_1V_1) = \frac{1}{1.67-1}\left(1\text{Pa}\cdot 1.5\text{m}^3 - 3\text{Pa}\cdot 1\text{m}^3\right) = -2.23881\text{J}$$

The work for the first adiabat $2 \to 3$ is:

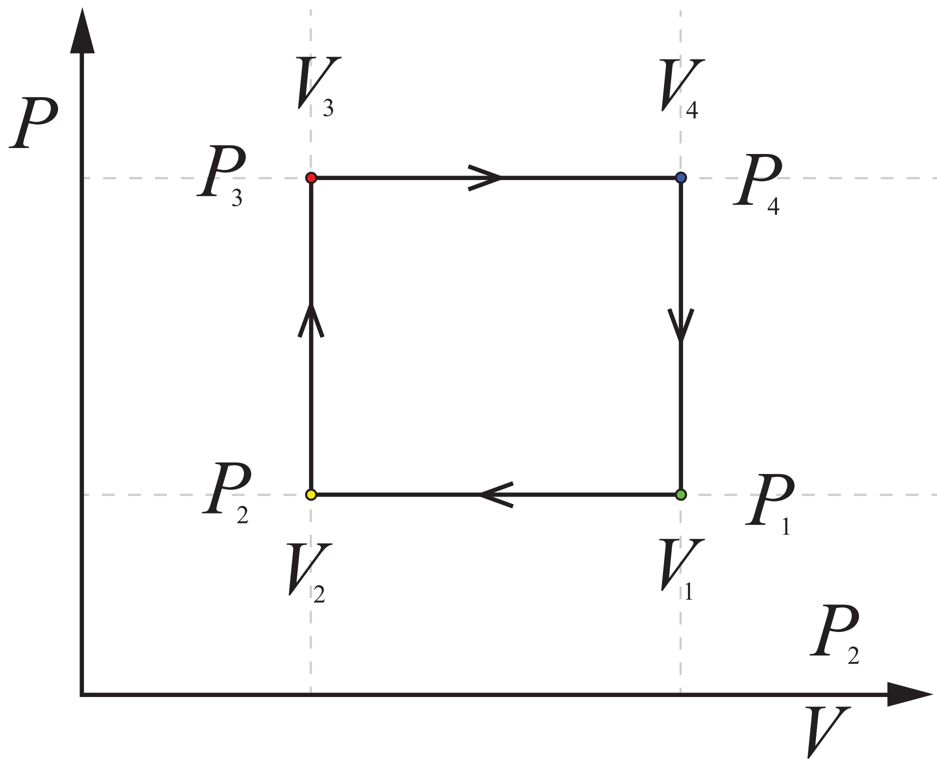
$$W_{\text{a2}} = \frac{1}{\gamma - 1}(P_2V_2 - P_3V_3) = \frac{1}{1.67-1}\left(1.5\text{Pa}\cdot 2\text{m}^3 - .5\text{Pa}\cdot 2.5\text{m}^3\right) = 2.61194\text{J}$$

So the total work for the carnot above is:

$$W_{\text{tot}} = W_{12} + W_{12} + W_{\text{a1}} + W_{\text{a2}}$$

$$= 40.3398\text{J} - 8.49401\text{J} - 2.23881\text{J} + 2.61194\text{J} = 32.2189\text{J}$$

This means the isochoric and isobaric must sum to 32.2189 Joules. A work cycle composed of isochoric and and isobaric processes looks like.



We know isochoric processes do know work so we just calculate:

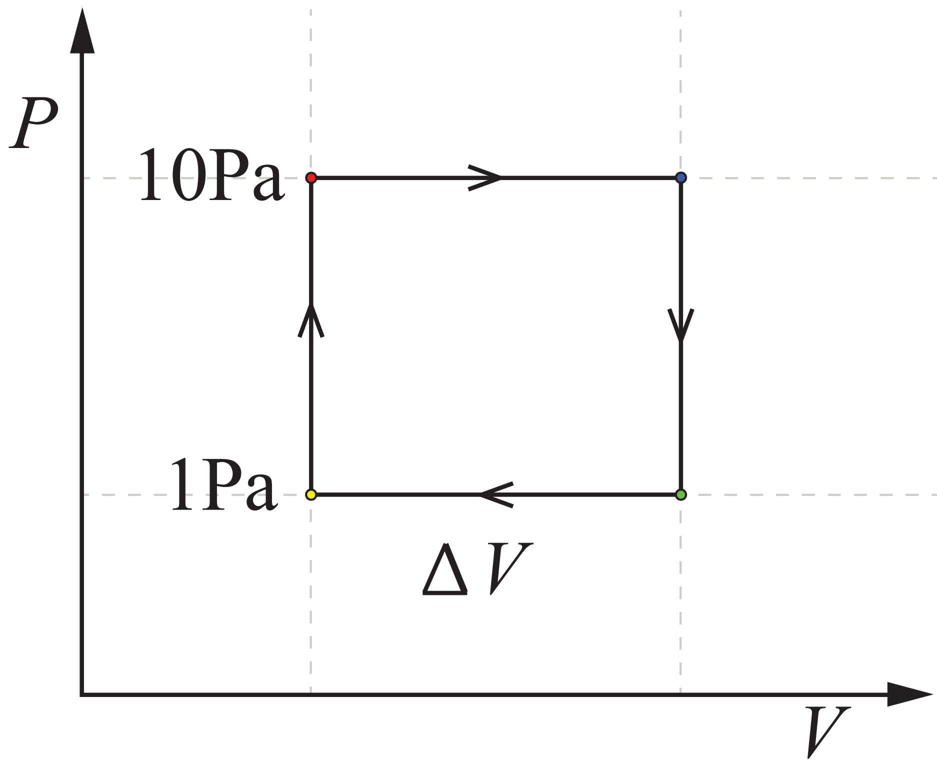
$$W_{\text{tot}} = W_{i1} + W_{i2} = -P_1 (V_2 - V_1) - P_3 (V_4 - V_3) = -1\text{Pa} (V_2 - V_1) - 10\text{Pa} (V_4 - V_3) = 32.2189\text{J}$$

Since $V_2 = V_3$ and$V_1 = V_4$ and the change in volume $\triangle V$ is the same for both isobars, our equation is reduced to

$$-1\text{Pa}\triangle V - 10\text{Pa}\triangle V = 32.2189\text{J}$$

$$11\text{Pa}\triangle V = -32.2189\text{J}$$

$$\triangle V = -\frac{32.2189\text{J}}{11\text{Pa}} = -2.92899\text{m}^3$$



The efficiency loss is:

$$e_{\mathrm{c}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}} = 1 - \frac{2}{7} = 0.714286\%$$

$$e_{\mathrm{i}} = 1 - \frac{Q_{\mathrm{c}}}{Q_{\mathrm{h}}} = 1 - \frac{1\mathrm{Pa} \cdot 2.92899\mathrm{m}^3}{10\mathrm{Pa} \cdot 2.92899\mathrm{m}^3} = .9\%$$

go figure