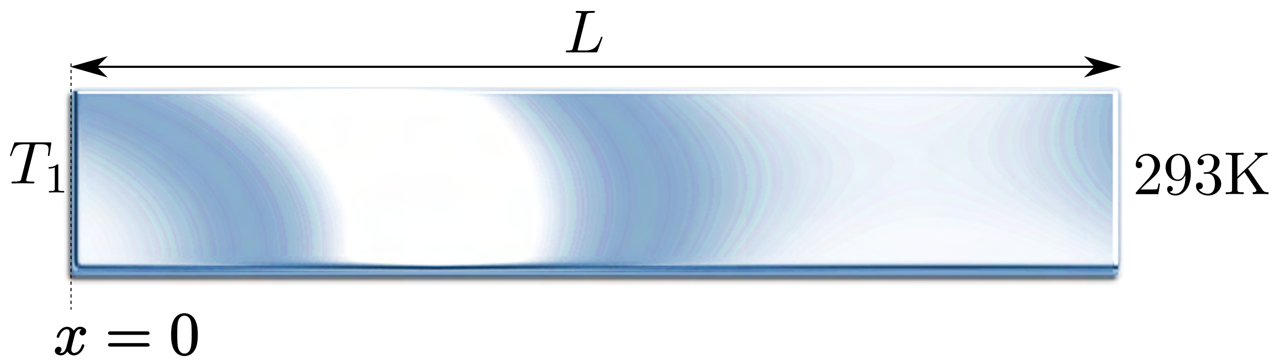
Thermodynamics - Sheet of metal with one side kept hot

March 29, 2012

Suppose you have a sheet of metal of stainless steel that is L long with a thermal diffusion coefficient $\kappa = 1\frac{\mathrm{m}^2}{\mathrm{s}}$ at 293K. One side is initially at T_1 and *all* the rest is intitally at 293K, but the other end must stay fixed at 293K.



1. Find $T(x,t)$ with a Gaussian ansatz. Plot the solution in 3-d.

Solution:

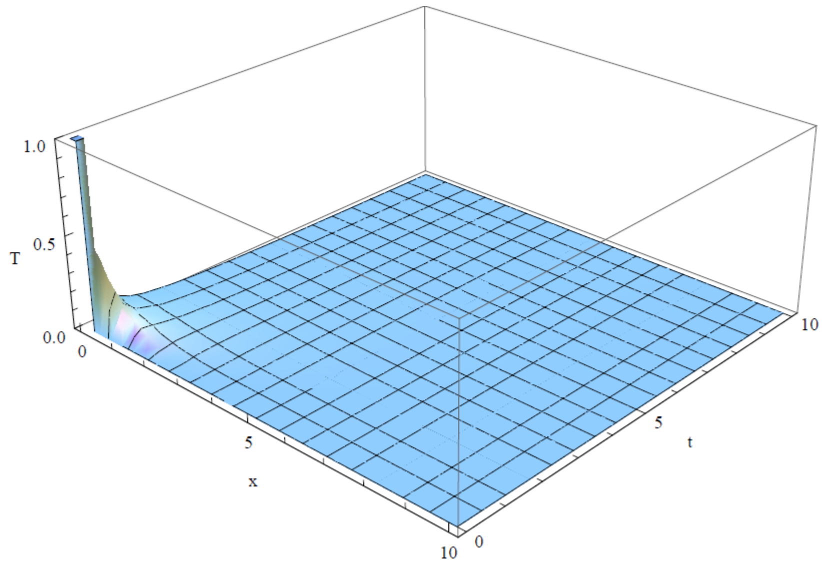
The diffusion equation is

$$\frac{\partial}{\partial t} T(x,t) = \kappa \frac{\partial^2}{\partial x^2} T(x,t)$$

The required solution is the Gaussian form:

$$T_{\mathrm{G}}(x,t) = \frac{1}{\sqrt{4\pi\kappa t}} \exp\left[-\frac{x^2}{4\kappa t}\right]$$

We may use the method of images as though the bar were $2L$ and $x=0$ was the center, then the solution has a condition $x>0$ and that will be $T(x,t)$. As we let $t \to 0$ we get a Dirac delta function, which is what we want for the left end. Therefore, given the initial conditions, $T(0,0) \simeq \delta(0)\, T_1$ and $T(L,0) = 273\mathrm{K}$ we may construct an analytical solution and construct 3-d plots on different scales to see the diffusion of heat take place.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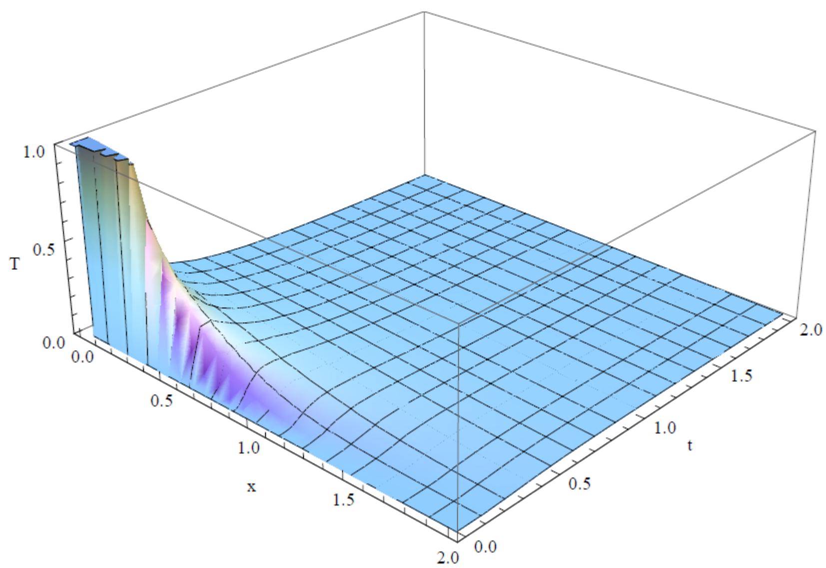
T
0.0
0
5
x
10
0
5
t
10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T
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x
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t
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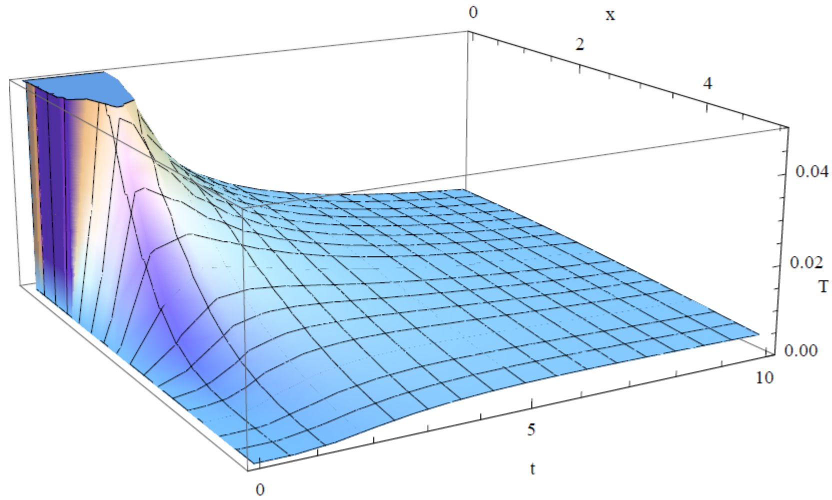

x
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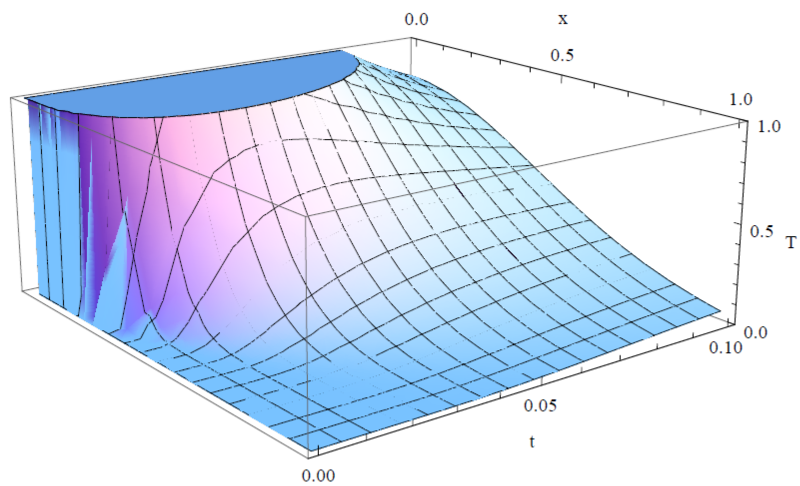
2
4
0.04
0.02
T
0.00
10
5
t
0



0.0
x
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1.0
1.0
0.5
T
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0.10
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0.00