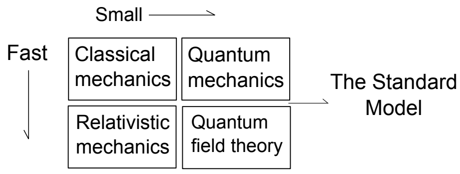
# 1. Resolving the 2nd Estate "Finite model"

PRIMER




The Standard model is for the fast and small. It is embodied in a non-abelian gauge field theory. It is a quantum field theory where the Lagrangian is invariant under transformations which form a Lie group referred to as the gauge group of the theory. Each group parameter has a corresponding vector field called gauge field which ensures Lagrangian gauge invariance.

Translation: *An incredibally complex mathematical model for particle physics*

The coupled ideas:

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$$

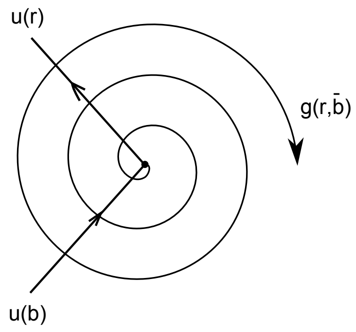
| Leptons | Quarks |
|---|---|
| $e$ | $u$ |
| $\nu_e$ | $d$ |
| $\mu$ | $s$ |
| $\nu_\mu$ | $c$ |
| $\tau$ | $t$ |
| $\nu_\tau$ | $b$ |

## 1.1. **Gauge Subgroups.**

$$SU_c(3)$$

Quantum Chromodynamics

Special Unitary group for Gauge bosons mediating the strong force. Gluons carry a color and an anti-color that they get from the 2 quarks they couple to as depicted below.




(see Appendix B)

That would mean we have

$$(rgb)^2 \rightarrow 9 \text{ gluons} \Longrightarrow \mathrm{r\bar{g}, r\bar{b}, r\bar{r}, g\bar{r}, g\bar{b}, g\bar{g}, b\bar{b}, \bar{b}g, b\bar{r}}$$

Due to color SU(3) symmetry these constitute a color octet and a color singlet. Confinement requires that all naturally occuring particles be color singlets, and this "explains" why the octet gluons never appear as free particles. Only the octets are considered in the Standard Model(at least observable, the ninth is speculated to be the photon). An octet is a multiplet with 8 fold degeneracy.

$$|1\rangle = \frac{r\bar{b}+b\bar{r}}{\sqrt{2}}, |2\rangle = \frac{-i(r\bar{b}-b\bar{r})}{\sqrt{2}}, |3\rangle = \frac{r\bar{r}-b\bar{b}}{\sqrt{2}}, |4\rangle = \frac{r\bar{g}+g\bar{r}}{\sqrt{2}},$$

$$|5\rangle = \frac{-i(r\bar{g}-g\bar{r})}{\sqrt{2}}, |6\rangle = \frac{b\bar{g}+g\bar{b}}{\sqrt{2}}, |7\rangle = \frac{-i(b\bar{g}-g\bar{b})}{\sqrt{2}}, |8\rangle = \frac{(r\bar{r}+b\bar{b}-g\bar{g})}{\sqrt{6}}$$

The Singlet has no purpose being here$\Longrightarrow |9\rangle = \frac{(r\bar{r}+b\bar{b}+g\bar{g})}{\sqrt{3}}$

*Note: Unlike photons, gluons can couple directly with themselves.*

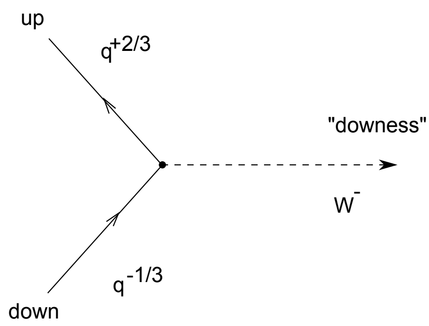
$$SU_L(2)$$

Weak Interactions

Special Unitary group for Gauge bosons mediating the weak force, $W^{\pm}$, $Z_0$. These have mass, a significant difference from the gauge bosons of the other groups. All quarks and all leptons interact with the weak force though. There are two kinds of weak interactions, charged ($W^{\pm}$) and nuetral ($Z_0$). Here is an example of a charged interaction with the emission of $W^{-}$gauge boson.

$$\mu^- + \nu_e \rightarrow e^- + \nu_{mu}$$

An example of an interaction of quarks with weak gauge boson:




(see Appendix B)

*Flavor is simply not conserved in weak interactions*

$U(1)$

Quantum electrodynamics

An abelian gauge theory with a symmetry group U(1) and one gauge field, the electromagnetic field. This is where Feynmann diagrams first appeared effective due to only small amount of vertices needed to describe an interaction or coupling. This is because a factor of $\alpha$ is introduced for every vertex. Diagrams with more and more vertices contribute less and less as follows.

$$\lim_{n\to\infty} \alpha^n = 0$$

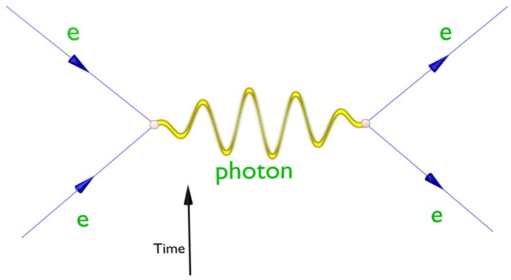
where

$$n = \text{vertices}$$

$$\alpha = \frac{e^2}{\hbar c 4\pi\epsilon_0} = \frac{1}{137.03599070}$$

(see Appendix B)

This is not so with QCD unless you're considering high energies exploiting "asymptotic freedom"(small distances or large energies) where the “running constant” becomes small. In QCD the coupling constant is a function of the energy or momentum. Here is a Feynman diagram of Moller Scattering, the general interaction of like charges with a common boson, the photon in this case.




(see Appendix B)

Bhabha scattering(symmetrizing unlike charges) can be seen by rotating the time vector by 90 degrees.

## 1.2. Fermions - Obedient to the exclusion people.

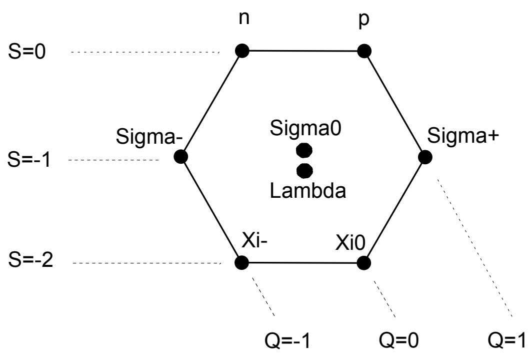
*Leptons:* [(e, $\nu_e$),($\mu$,$\nu_\mu$), ($\tau$, $\nu_\tau$)]

These are subject to 4 gauge bosons $\gamma, W^-, W^+, Z_0$ only. They get heavier with every generation. Tau is about $1,776.84 \pm 0.17 MeV/c^2$ (compared to $938\text{MeV}/c^2$ for protons and $0.511\text{MeV}/c^2$ for electrons). Interestingly their mean lifetime is about $2.906 \times 10^{-13}\text{s}$(B1). This is due to the Uncertainty princple with relativistc particles being $\Delta E \Delta t > h$.

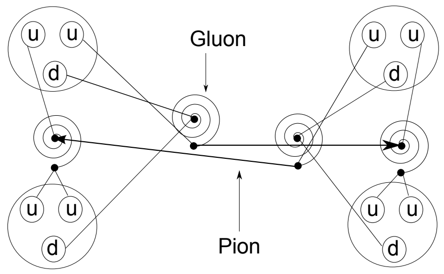
*Quarks:* 6 flavors $[(d,u),(s,c),(t,b)]$

Subject to all gauge bosons. Each of these carry one of three colors; red, green or blue. Three of them together form the familiar nucleons, protons and neutrons which are Baryonic. Again, they get heavier with every generation. The Top quark weighs in at $171.2 \pm 2.1\text{GeV}/c^2$. The Standard Model predicts its lifetime to be roughly $1 \times 10^{-25}\text{s}$; this is about 20 times shorter than the timescale for strong interactions, and therefore it does not hadronize. This is where strangeness first appears which is only conserved in weak interactions.

One representation of the eight lightest Baryons is the Baryon Octet



Baryons stuck together by their friend the $\pi^0$meson.




(see Appendix B)

There are 3 pions(mesons) $\pi^+$,$\pi^0$ and $\pi^-$, the above is a Pion zero.

At least one scalar Higgs multiplet is needed to give the vector bosons and fermions their masses. To see this we need some rich math.

The Euler-Lagrange equations of motion:

$$L = \sum_i L_i(q_i, \dot{q}_i) \to \int \mathcal{L}(\phi_k, \partial^\alpha \phi_k) d^3x$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) = \frac{\partial L}{\partial q_i} \to \partial^\beta \frac{\partial \mathcal{L}}{\partial(\partial^\beta \phi_k)} = \frac{\partial \mathcal{L}}{\partial \phi_k}$$

Where $\mathcal{L}$ is the Lagrangian density

These come from the action integral:

$$A = \int \int \mathcal{L} d^3x dt = \int L d^4x$$

(see Appendix A)

For U(1) the Lagrangian density is postulated to be:

$$\mathcal{L} = -\frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} - \frac{1}{c} J_\alpha A^\alpha$$

The two terms represent the field and the interaction, which we can restate as:

$$\mathcal{L} = -\frac{1}{16\pi} g_{\lambda\mu} g_{\nu\sigma} (\partial^\mu A^\sigma - \partial^\sigma A^\mu)(\partial^\lambda A^\nu - \partial^\nu A^\lambda) - \frac{1}{c} J_\alpha A^\alpha$$

Placing this into the equation of motion we get:

$$\frac{1}{4\pi} \partial^\beta F_{\beta\alpha} = \frac{1}{c} J_\alpha$$

This is the covarint form of the inhomogeneous Maxwell's equations, to get the others we use the dual field strength tensor:

$$\mathcal{F}^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} F_{\gamma\delta}$$

## Introducing the Standard Model Lagrangian

(see Appendix C)

$$L_{SM} = L_{YM} + L_{WD} + L_{Y_u} + L_H$$

First Term:

$$L_{YM} = L_{QCD} + L_{I_w} + L_Y$$

$$= -\frac{1}{4g_3^2}\sum_{A=1}^{8} G^A_{\mu\nu}G^{\mu\nu A} - -\frac{1}{4g_2^2}\sum_{a=1}^{3} F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{4g_1^2}B_{\mu\nu}B^{\mu\nu}$$

This part describes the low energy gauge groups of the standard model, $SU(3)$ for color, $SU(2)$ for weak isospin and $U(1)$ for hypercharge. The "running constants" $g_i, i = 1,2,3$ are what allows asymptotic freedom at high energies.

Second Term:

$$L_{WD} = \sum_i^3 \left( L_i^\dagger \sigma^\mu D_\mu L_i + \bar{e}_i^\dagger \sigma^\mu D_\mu \bar{e}_i + \mathbf{Q}_i^\dagger \sigma^\mu D_\mu \mathbf{Q}_i + \bar{u}_i^\dagger \sigma^\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \sigma^\mu D_\mu \bar{d}_i \right)$$

This part describes the fermion fields and their gauge interactions. The “particles” are found in this term as follows

Weak doublet of leptons: $L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \to (\mathbf{2}, \mathbf{1}^c)_{y_1}$

Weak singlet of a lepton: $\bar{e}_{iL} = (\mathbf{1}, \mathbf{1}^c)_{y_2}$

Weak doublet of quarks: $\mathbf{Q}_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \to (\mathbf{2}, \mathbf{3}^c)_{y_3}$

Weak singlets of an antiquark: $\bar{\mathbf{u}}_{iL} \to (\mathbf{1}, \bar{\mathbf{3}}^c)_{y_4}$

Weak singlets of an antiquark: $\bar{\mathbf{d}}_{iL} \to (\mathbf{1}, \bar{\mathbf{3}}^c)_{y_5}$

Let’s look at a few of the terms from $L_{WD}$, first the Covariant derivative with respect to the gauge group of the weak doublet of leptons

$$D_\mu L_i = (\partial_\mu + iW_\mu + \frac{i}{2}y_1 B_\mu)L_i$$
$$\partial_\mu = (\tfrac{\partial}{\partial x^0}, \nabla)$$
$$\partial_\mu \mathbf{Q_i} = (\tfrac{\partial}{\partial x^0}, \nabla)\mathbf{Q}_i$$

The first part on the R.H.S. is the lepton’s own field while the other two represent the coupling to the vector bosons($W^\pm, Z_0, \gamma$). The familiar Pauli matrices are hidden in $\mathbf{W}_\mu$

Now for another, the quark weak doublet term

$$D_\mu \mathbf{Q}_i = (\partial_\mu + i\mathbf{A}^*_\mu iW_\mu + \frac{i}{2}y_3 B_\mu)\mathbf{Q}_i$$

Let’s look at some of the terms in this covariant derivaive

$$\mathbf{A}_\mu = \frac{1}{2}A^A_\mu(x)\lambda^A$$

$$D_\mu L_i = (\partial_\mu + iW_\mu + \frac{i}{2}y_1 B_\mu)L_i$$

Let’s look at $A = 3$

$$\lambda_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This is a Gell-Mann matrix, an infinitesmal generator of the $SU(3)$ group. Notice that it’s unitary with a Trace of 0.

Third Term:

$$L_{Y_u} = i(\hat{L}_i\bar{e}_j + \hat{Q}_i y^{[d]}_{ii}\bar{e}_i)H^* + i\hat{Q}_i U_{ji} y^{[u]}_{ii}\bar{u}_j\tau_2 H + c.c.,$$

This is where the Yukawa couplings are introduced.

It is remarkable that one of these solutions is realized by the Yukawa couplings. Under the $SU_2^w \times U_1^Y$ gauge group, potential color singlet quark masses violate weak isospin by half units, i.e. $\Delta I_W = \frac{1}{2}$. In order to generate masses for the charged fermions, withou violating the renormalizability of the theory, on introduces a spinless boson Higgs field $H$, transforming as a weak doublet, color singlet, and with hypercharge $y_n$.

At last:

$$L_H = (D_\mu H)^\dagger (D_\mu H) - V(H)$$

Where

$$D_\mu H = (\partial_\mu + iW_\mu + \frac{i}{2} y_h B_h) H$$

and

$$H = \begin{matrix} H_1 \\ H_2 \end{matrix} \sim (\mathbf{1}^c, \mathbf{2})_{y_h}$$

"A proton is a nucleon is a hadron is a baryon is a composite fermion is a..."

## APPENDIX

*A. Action to Motion.*

Say we have a functional

$$J(\alpha) = \int_{x_1}^{x_2} f\{y(\alpha, x), y'(\alpha, x); x\} dx$$

where $y(\alpha, x) = y(0, x) + \alpha\eta(x)$

Now vary the functional with a functional derivative

$$\frac{\partial}{\partial\alpha} J(\alpha) = \frac{\partial}{\partial\alpha} \int_{x_1}^{x_2} f\{y(\alpha, x), y'(\alpha, x); x\} dx$$

$$\frac{\partial}{\partial\alpha} J(\alpha) = \int_{x_1}^{x_2} (\frac{\partial f}{\partial y} \frac{\partial y}{\partial\alpha} + \frac{\partial f}{\partial y'} \frac{\partial y'}{\partial\alpha}) dx$$

$$\frac{\partial}{\partial\alpha} J(\alpha) = \int_{x_1}^{x_2} (\frac{\partial f}{\partial y} \eta(x) + \frac{\partial f}{\partial y'} \frac{\partial\eta(x)}{\partial\alpha}) dx$$

Integrate the 2nd term by parts

$$\int_{x_1}^{x_2} \frac{\partial f}{\partial y'} \frac{\partial\eta(x)}{\partial\alpha} dx = \frac{\partial f}{\partial y'} \eta(x)|_{x_!}^{x_2} - \int_{x_1}^{x_2} \frac{d}{dx} \frac{\partial f}{\partial y'} \eta(x) dx$$

The integrated term vanishes due to the requirement of

$\eta(x_1) = \eta(x_2) = 0$.

$$\frac{\partial}{\partial\alpha} J(\alpha) = \int_{x_1}^{x_2} [\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'}] \eta(x)] dx$$

Since $\eta(x)$ is an arbitrary function and we want an extremum we arrive at

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0$$

Field Theory

B. Feynmann calculus. In a Feynmann diagram there are rules for calculating the amplitude $\mathcal{M}$ of that set of vertices, the Feynmann rules. This involves 6 steps.

*1. Notation.* Label external lines with $p_1, p_2, ....$ , and internal lines with $q_1, q_2, ....$

*2. Coupling constant.* For each vertex write a factor $-ig$, where g is the coupling constant(strength of interaction)

*3. Propogator.* For each internal line write a factor $\frac{i}{q_j^2 - m_j^2 c^2}$. The mass of the particle enters here.

*4. Conservation of Energy and Momentum.* For each vertex write a delta function of the form $(2\pi)^4 \delta^4(k_1 + k_2 + k_3)$. The k's are for the external lines. This factor imposes conservation of energy and momentum at each vertex, since the delta function is zero unless the sum of the incoming momenta equals the sum of the outgoing momenta.

*5. Integration of internal Momenta.* For each line write down a factor
$\frac{1}{(2\pi)^4} d^4 q_j$
and integrate over all internal momenta.

*6. Remove delta function.* From here if you want cross sections or life times you would plug the $\mathcal{M}$ into
$\frac{d\sigma}{d\Omega} = (\frac{\hbar c}{8\pi})^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$
$\Gamma = \frac{S|\mathbf{p}|}{8\pi \hbar m_1^2 c} |\mathcal{M}|^2$

*C. Field Strengths.*

The color field strength for 8 gluon fields:

$$G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - f^{ABC} A^B_\mu A^C_\nu, \quad A, B, C = 1, ..., 8,$$

$A^B_\mu$ are the eight gluon fields and $f^{ABC}$ are the structure functions of $SU(3)$

Example from Classical Electrodynamics for $U(1)$ where we only have $A = 1$ with trivial structure functions:

$$A^\alpha = (\Phi, \mathbf{A})$$

$$\mathbf{E} = -\frac{\mathbf{1}}{\mathbf{c}} \frac{\partial \mathbf{A}}{\partial \mathbf{t}} - \nabla \mathbf{\Phi}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$$

$$E_x = -\frac{1}{c} \frac{\partial \mathbf{A_x}}{\partial \mathbf{t}} - \nabla_x \Phi = -(\partial^0 A^1 - \partial^1 A^0)$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = -(\partial^2 A^3 - \partial^3 A^2)$$

These equations imply that the electric and magnetic fields, six components in all, are the elements of a second rank, antisymmetric field-strength tensor,

$$F^{\alpha\beta} = (\partial^\alpha A^\beta - \partial^\beta A^\alpha) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & 0 & B_y \\ -E_y & 0 & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

DC 131:7 Abraham 3:26 DC 88: 67

# References

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[6]